State-Space Models with Kalman Filtering for Freeway Traffic Forecasting

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Abstract

A challenge in road traffic operations and management is that it is not possible to process data in real-time and use the output in control algorithms. This is due to the fact that by the time data are processed and a control measure applied, the traffic will have passed. A solution to this is to predict the traffic state based on assessments of current and past measurements. In the work described in this paper, forecasting the state of traffic volume was achieved using a Kalman filter that was employed in a dynamic state-space model framework where parameters of the state are permitted to change with time.

Introduction

The need for forecasting with the Kalman filter arose from a challenge in processing real-time traffic data. This is due to the fact that by the time processing is completed and a control measure applied, the traffic will have passed. A solution to this problem is to forecast the traffic state and implement a control measure based on the forecast.

The state-space framework considers a time series as the output of a dynamic system perturbed by random disturbances and ones in which parameters are allowed to vary over time [1]. A special case of general state-space models that are linear and Gaussian are also called dynamic linear models. A Kalman filter is an optimal recursive data processing algorithm, meaning that predictions are based on only the previous time-step's prediction and the filter does not require all previous data to be stored and reprocessed with new measurements. The filter is optimal in the sense that it minimizes the variance of the estimation error at each iteration process. A Kalman filter was used to analyze traffic data to make predictions on traffic volume for Interstate-84 (I-84) in Meridian, Idaho.

The Kalman filter works by making a prediction of the future and comparing the estimate with real-time measurements. Along with the prediction, an error covariance is calculated. When the next measurement is taken, the algorithm calculates a correction of the state prediction using the new measurement along with the error covariance. The recursive algorithm uses only the current measurement and error covariance allowing for low computational cost and on-line forecasting.

ISBN 978-1-60643-379-9

High-resolution (5-minute aggregate) data—volume, occupancy, vehicle classification, and average lane speed—were collected on I-84 near the Meridian Road and Eagle Road interchanges. Six radar based (SmartSensor10 from Wavetronix) sensing devices were chosen to capture the data from November 7-15, 2013.

Volume data (depicted in Figure 1) for I-84 mainline, prior to the Eagle Road eastbound loop on-ramp, was of interest to forecast. Weekend observations were removed as typically weekend traffic volume is significantly lower and is not of interest to forecast at this time. The work reported here on traffic forecasts is a part of a bigger project, which will use the traffic forecasts in the control of on-ramp volumes through metering.

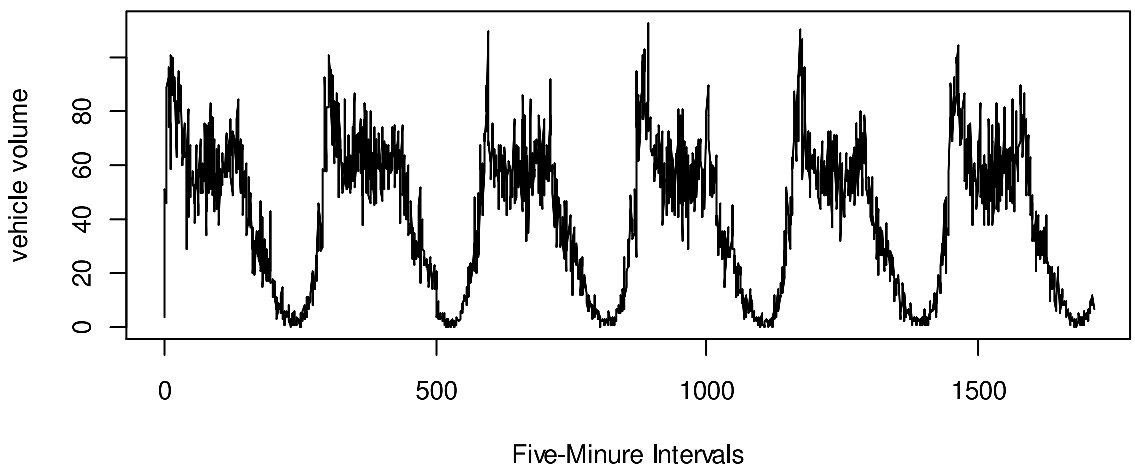

Figure 1. Five-minute average volumes with weekend data removed

State-Space Framework

State-space models can be used for modeling univariate non-stationary time series that allow for natural interpretation as a result of trend and seasonal (periodic) components [1, 2]. A local level model is a time series where observations can be modeled as random fluctuations around a stochastic level (described by a random walk). An extension to the local level model is one with linear trend and seasonal components. A stochastic local level model with a seasonal component was constructed and its parameters estimated by maximum-likelihood estimation (MLE) in the “R” language and environment for statistical computing [3].

The main tasks for the given state-space model were to make inference on the unobserved traffic state and predict future observations based on part of the observation sequence. Estimation and forecasting are solved by computing conditional distributions of the traffic state, given the available information. In dynamic state-space models, the Kalman filter provides the formulas for updating our current inference on the state vector.

Kalman Filter

The general problem that the Kalman filter addresses is the estimation of the state x_t of a discrete-time controlled process that is governed by the general state equation [4]:

$$x_t = G_t x_{t-1} + w_t \tag{1}$$

based on measurements y_t according to the observation equation:

$$y_t = F_t x_t + v_t \tag{2}$$

where G_t and F_t are known matrices and v_t and w_t are independent white noise sequences with $w_t \sim \mathcal{N}(0,Q_t)$ and $v_t \sim \mathcal{N}(0,R_t)$.

The Kalman filter can be thought of as a recursive two stage prediction and measurement update algorithm with the prediction stage equations given by:
State estimate (*a priori*):

$$\hat{x}_{t|t-1} = G_t \hat{x}_{t-1|t-1} \tag{3}$$

Error covariance estimate (*a priori*):

$$P_{t|t-1} = G_t P_{t-1|t-1} G_t^T + Q_t \tag{4}$$

The predicted state estimate is also known as the *a priori* state estimate because it does not include information from the current time step. In the measurement update stage, the prediction is combined with the current observation information to refine the state estimate and is known as the *a posteriori* state estimate. The measurement update equations are given by

Measurement innovation:

$$\tilde{y}_t = y_t - F_t \hat{x}_{t|t-1} \tag{5}$$

The innovation covariance:

$$S_t = F_t P_{t|t-1} F_t^T + R_t \tag{6}$$

Kalman filter gain:

$$K_t = P_{t|t-1} F_t^T S_t^{-1} \tag{7}$$

State estimate (*a posteriori*)

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t \tilde{y}_t \tag{8}$$

Error covariance estimate (*a posteriori*):

$$P_{t|t} = P_{t|t-1} - K_t F_t P_{t|t-1} \tag{9}$$

The Kalman filtering state matrix:

$$G_t = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{10}$$

With noise covariance:

$$Q_t = \begin{bmatrix} 12.01 & 0 \\ 0 & -1 \end{bmatrix} \tag{11}$$

Observation design matrix

$$F_t = \begin{bmatrix} 1 & 1 \end{bmatrix} \tag{12}$$

and observation noise:

$$R_t = [58.75] \qquad (13)$$

A Kalman filter computed one-step ahead values for the state vector, together with their variance/covariance matrices.

A Kalman smoother computed smoothed values of the state vectors together with their variance/covariance matrices. The smoothing algorithm estimates the state sequence at times 1,...,t, given the data $y_1,\ldots y_t$, by a recursive algorithm.

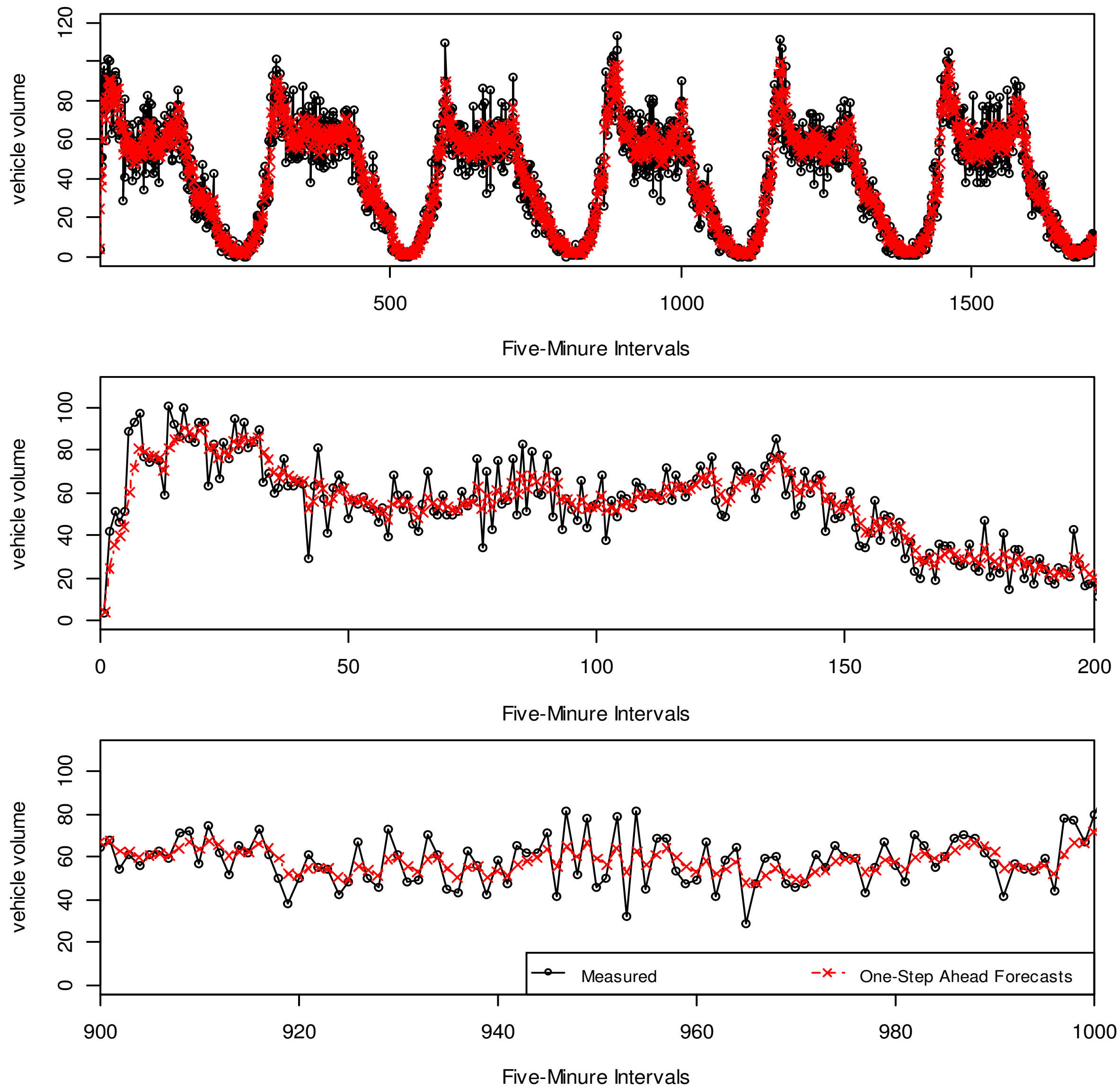

Figure 2. Measured traffic state with one-step ahead forecasts, shown in detail in second and third plots

As seen in Figure 2, the Kalman filter had a “burn-in” time through about 25 five-minute observation periods. The filter’s one-step ahead forecasts appear to produce good predictions after this period.

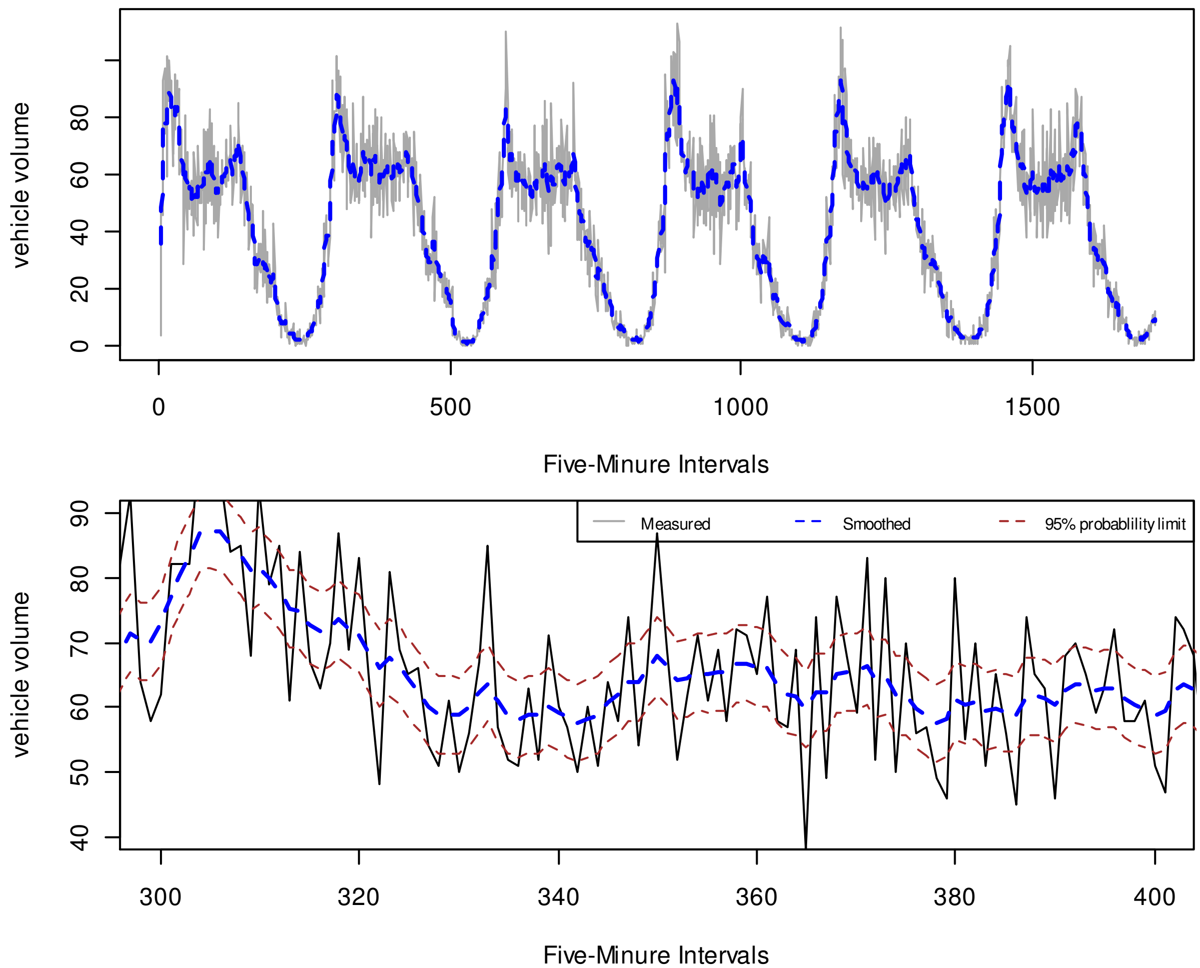

Figure 3. Smoothed state estimates, shown in detail in second plot

The Kalman smoother’s output is shown in Figure 3. The smoothing algorithm is not an ideal solution when online processing is required, since it uses the sequence of observations up to the current observation. It can, however, provide more reliable estimates by using additional measurements made after the time of the estimated state.

Model Validation

The model was validated using ramp flows from Eagle Road interchange eastbound on-ramp from one day of observations from 6 AM-12PM. A smaller data set was chosen with larger deviations in observations to test the Kalman filters “tuning” time and performance. It should be noted that the model was calibrated against the five-minute traffic volume data and validated against the hourly traffic flows.

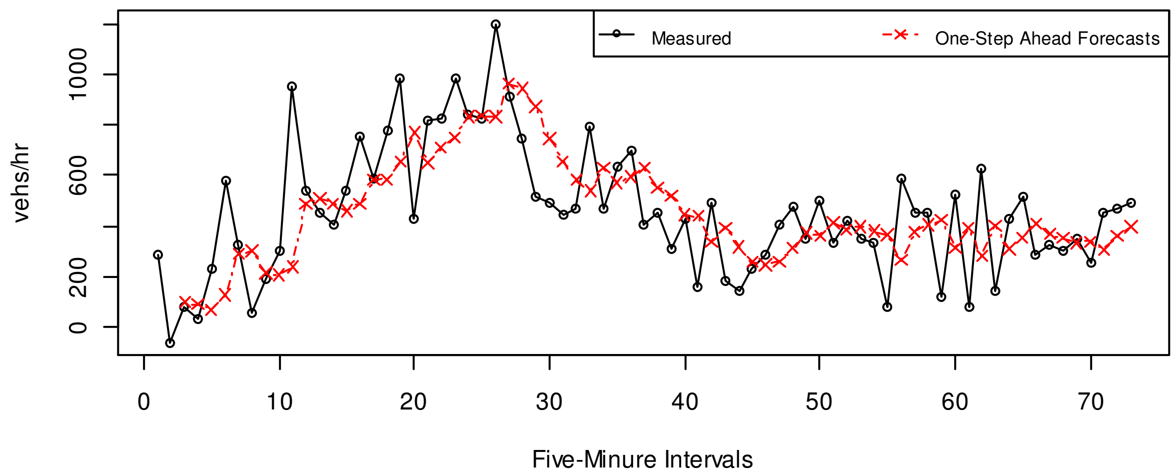

Figure 4. Eagle Road on-ramp one-step ahead flow predictions

As seen in Figure 4, the model's one-step ahead forecasts appeared to converge to reasonable estimates relatively early. Note that at the second time step, sensors recorded a negative flow, and the Kalman filter was able to process the noisy measurement. Smoothed estimates from the Kalman filter are shown in Figure 5.

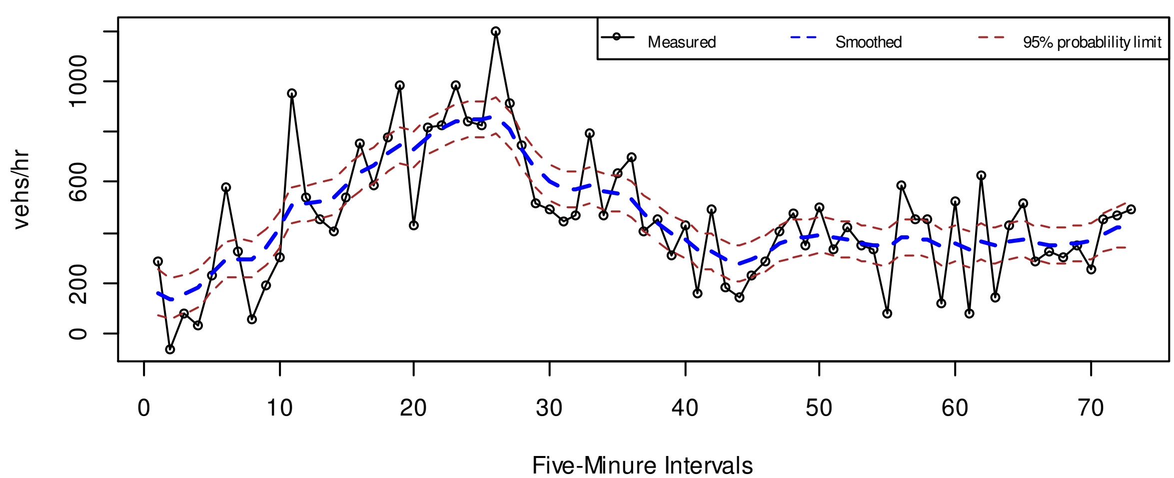

Figure 5. Eagle Road on-ramp smoothed estimates

Conclusion

The present model is based on the idea that the observations y_t for traffic volume are incomplete and a noisy function of the unobservable state process x_t. The unobservable state process is the actual traffic volume. We can only observe this through noisy measurements and are modeling an abstraction of it.

The traffic volumes observed on I-84 are characterized by a mixture of smooth gradual changes over a day as well as rapid fluctuations that occur during a twice-daily more congested period. To capture these dynamics, the traffic state was modeled in a state-space framework with its predictions performed by a Kalman filter. A polynomial state-space model with stochastic local level and seasonal components parameters were estimated by the maximum likelihood estimation (MLE) method. The uncertainty associated with the MLE's standard errors were 6.11 and 10.70. Plots of the Kalman filters output appeared that it "tuned" relatively early and had good performance.

The state-space equations should be formulated to represent the system being observed. Since this is not always possible, the evolution of the predictions from the Kalman filter may not be the minimum error variance. However, the method described in this paper can be used in many areas where estimation of the traffic state is needed such as dynamic traffic management and control. When applied to travel speeds, this methodology may provide real-time freeway travel time predictions. A real-time, short-term traffic flow forecasting was presented in this paper. Such a procedure can be incorporated into an adaptive ramp metering scheme for better traffic management.

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Biographies

BRIAN PORTUGAIS received his B.S. in Civil Engineering from Boise State University in 2012. Currently, he is working as a graduate research assistant at Boise State University pursuing a M.S. in Civil Engineering.

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